Optimal Inflation Rates: A Generalization

A Note by Alvin L. Marty and Frank J. Chaloupka

In his classic article, "The Welfare Cost of Inflationary Finance," (1956), Bailey indicted the use of the printing presses to raise resources on the grounds that the average ratio of the welfare cost to the revenue becomes excessive at moderate rates of inflation. He generalized these results to a competitive banking system paying interest on its deposits, but subject to a sterile legal reserve requirement. Any single bank would be forced by competition to pay interest on its deposits equal to the yield on its assets times $(1-f)$, where f is the legal reserve ratio. The opportunity cost of holding deposits is then the return on assets minus the yield on deposits which, in turn, is equal to the yield on assets times f. Bailey assumed the real rate was zero (not a crucial assumption) and that at high enough rates of inflation no one would use currency, so that all money would be held as interest bearing deposits.[1] The opportunity cost of holding deposits would then be the actual (and anticipated) rate of inflation, π, times the reserve ratio, while the seignorage accruing to the authorities would be $\pi f M/P$, the return on assets, $\pi M/P$, multiplied by the ratio of high-powered money to the money supply. Bailey showed that to get the same tax revenue as in a pure currency system $(f=1)$, the authorities must inflate at a much higher rate, namely, the rate of inflation (when $f=1$) times the reciprocal of the reserve ratio $[\pi(1/f)]$. A reduction in the reserve ratio raises the inflation rate needed to command the same resources (a proposition rediscovered by Calvo and Fernandez [1983]). Al-

Alvin L. Marty thanks the City University Research Foundation for an F.R.A.P. award supporting this research.

[1]If currency and deposits are perfect substitutes, the payment of any interest on deposits would cause currency in private hands to go to zero. This is not our reading of Bailey. Rather, currency and deposits are imperfect substitutes and what determines the currency-deposit ratio is the difference between the opportunity costs of holding currency and deposits $(i-if)$. At very high interest rates this difference becomes substantial enough to induce individuals to use only deposits.

Recently, Dwyer and Saving (1976) suggest that the currency-deposit ratio depends on the ratio of their opportunity costs (i/if), and if the money demand function is homothetic in currency and deposits, the currency-deposit ratio is independent of the rate of interest at all levels of real balances. We do not find this reasoning compelling since it implies that individuals would not alter their holdings of currency and deposits if the *difference* in the opportunity costs continued to rise while the ratio remained constant.

ALVIN L. MARTY *is professor of economics at Baruch College and the Graduate Center of the City University of New York.*
FRANK J. CHALOUPKA *is a Ph.D. candidate at the Graduate Center.*

Journal of Money, Credit, and Banking, Vol. 20, No. 1 (February 1988)

though the inflation rate is higher by the reciprocal of the reserve ratio, the payment of interest on deposits induces individuals to hold the same real cash balances as when $f=1$ and the government commands the same flow of resources. Although the tax rate, π/f, is higher, the tax base, $(M/P)f$, is now proportionately lower so that, as Bailey noted, the *average* ratio of the welfare cost to revenue remains unchanged. Clearly all these propositions go through if we drop the assumption that the real rate is zero and, if following Aurenheimer (1974) and Phelps (1973), the tax rate is taken as the money rate of interest, so that the revenue is now $if(M/P)$. Thus, if the money rate is 5 percent when $f=1$, a money rate of 10 percent would be necessary if the reserve ratio is reduced to ½. However, the opportunity cost of holding real balances would remain 5 percent.

Matters become somewhat more interesting if, in line with modern differential tax analysis, we ask whether the Bailey propositions carry over to the ratio of the marginal welfare cost to the marginal increment to tax revenue $(\delta W/\delta R)$. A moment's reflection indicates that the Bailey results do carry over to the marginal ratios. A short proof may be in order (presented some years ago by the senior author with $f=1$, [Marty 1976]). The revenue is $R=(if)(M/P) = (if)\phi(if)$ and the Welfare loss is $W=\int_0^{if} \phi(x)dx - (if)\phi(if)$.

$\delta W/\delta(if)=-(if)\phi'(if)$ and $\delta R/\delta(if)=(if)\phi'(if) + \phi(if)$, so that $\delta W/\delta R= \eta_{if}/(1-\eta_{if})$, where the elasticity of demand for real balances $\eta_{if}=-(if)\phi'(if)$.

Suppose now we set $\delta W/\delta R=MC$ where MC is an index of the marginal welfare loss to revenue of other distorting taxes. This represents an application of the inverse elasticity rule of Ramsey, a special rule requiring all cross effects within the taxed sector to be nonexistent (Marty 1978). In turn, real balances are boldly treated as a final commodity produced at zero marginal cost.

When competition forces a bank to pay explicit interest $i(1-f)$ on its deposits, the Ramsey rule requires that the money rate of interest equal that "optimal" rate which would have been set if all money were fiduciary currency ($f=1$), multiplied by the reciprocal of the reserve ratio. $\delta W/\delta R$ then remains equal to the fixed MC. The authorities, in this case, collect the same revenue and incur the same average and marginal welfare loss as in a pure currency system.

We now turn to the case of nonprice competition. Suppose regulation prohibited the payment of explicit interest on deposits. Such regulation probably did not limit the extent of competition but altered its nature to nonprice dimensions. Suppose subsidies are less useful to depositors than interest income. Although competition forces any individual bank to pay $i(1-f)$ in subsidized services, the depositor values these at only $ki(1-f)$ where $k<1$. The net opportunity cost of holding deposits is then $i-ki(1-f)$. Let $Z\equiv i-ki(1-f)$. The tax revenue is $R=(if)\phi(Z)$, and the welfare loss is $W=\int_0^Z \phi(x)dx-Z\phi(Z)$. It follows that $\delta W/\delta R=\eta_{if}/(1-\eta_Z)$ where η_{if} is the elasticity of demand with respect to (if) and η_Z is the elasticity of demand with respect to Z.[2] In this case of nonprice competi-

[2]Given R and W as defined above, $\delta R/\delta(if)=(if)\phi'(Z)\delta Z/\delta(if)+\phi(Z)$ and $\delta W/\delta(if)= -Z\phi'(Z)\delta Z/\delta(if)$. Then $\delta W/\delta R=-Z\phi'(Z)[\delta Z/\delta(if)]/[\phi(Z)+(if)\phi'(Z)\delta Z/\delta(if)]$. Since $\delta Z/\delta(if)=$

tion, for a given MC, both the revenue collected and the "optimal" inflation rate are lower than when deposits pay explicit interest ($k=1$).

To illustrate the importance of keeping in mind these general formulas, consider the recent note by McClure (1986). He does not present the general formulas but rather derives specific cases using the Cagan demand function, $M/PY=\alpha e^{-\beta(\pi+r)}$. Different monetary institutions are assumed: one makes the Bailey assumption that a competitive bank pays interest on deposits of $i(1-f)$ so that the opportunity cost of holding real balances is (if). McClure initially sets the marginal cost of other taxes at 0.1, the real rate r at 0.05, and the semilog slope of the demand function β at 0.5. The general formula would have $MC=\eta_{if}/(1-\eta_{if})$. Since the elasticity of the Cagan function is $(if)\beta$, $MC=if\beta/(1-if\beta)$. If $f=1$, $i^*=18.18$ percent, or $\pi^*=13.18$ percent. Now assume that $f=0.4$ instead of 1. Clearly, the previous interest rate must be multiplied by the reciprocal of the new reserve ratio, 2.5. The new optimal $i^{**}=45.45$ percent and the $\pi^{**}=40.45$ percent, almost three times that calculated by McClure. When $MC=0.5$ and $f=1$, the optimal $i^*=66.67$ percent and the $\pi^*=61.67$ percent. When the reserve ratio is 0.4, the optimal i^{**} is now 166.675 percent and the π^{**} is 161.675 percent, almost twice as large as the π^{**} of 85.91 percent reported by McClure.[3]

Finally, consider the unrealistic case in which an effective prohibition exists on the payment of interest on demand deposits. McClure's calculations are, in this case, correct. Nevertheless, it is preferable to present general formulas thereby facilitating the application of specific demand functions. The general proof is precisely the same as previously derived, $\delta W/\delta R=[\eta_i/(1-\eta_i)]/f$. At any money rate of interest, the welfare costs are independent of f. However, when $f<1$, the banks appropriate a portion of the seignorage and the authority's share is reduced by the ratio of high-powered money to the money stock. Since the marginal welfare costs is independent of f and the marginal increment to revenue is reduced when $f(1$, $\delta W/\delta R$ is raised, perhaps considerably. The optimal elasticity of demand and the optimal money rate are reduced as shown by the general formula $f(MC)=\eta_i/(1-\eta_i)$.

In conclusion, it is important to keep in mind both the limitations and possible extensions of our analysis. First, cash balances have been treated as a final good

---

$Z/(if)$ and defining the elasticity of demand for balances with respect to Z as $\eta_Z=-Z\phi'(Z)/\phi(Z)$ and the elasticity with respect to (if), $\eta_{if}=-[Z\phi'(Z)\delta Z/\delta(if)]/\phi(Z)$, we have $\delta W/\delta R=\eta_{if}/(1-\eta_Z)$.

It may be helpful to note that the opportunity cost of holding deposits in the case of nonprice competition, $Z\equiv i(1-k)+kif$, is a weighted average of the opportunity costs under a pure currency regime (i) and one in which explicit interest is paid (if).

We are indebted to an anonymous referee for suggesting this extension to the case of nonprice competition.

[3]McClure's mathematical slip occurs in his equation (9). Our general formulas when applies to the Cagan function yield $\delta W/\delta(if)=-if\beta\alpha e^{-\beta if}$. Since $\delta W/\delta\pi=(\delta W/\delta(if))f$, McClure's $\delta W/\delta\pi$ (equation 9) should have been multiplied by f.

McClure adds to the traditional marginal welfare cost the term $Z(M/PY)$, so that the welfare loss due to a marginal decrease in real balances is measured by the money rate of interest plus Z. What precisely Z is and how it can be quantified is left up in the air. We have therefore chosen to ignore this catchall variable and leave it to the interested reader to correct the remainder of his table 2.

produced at zero marginal cost and have been entered directly as an argument in the utility function. However, if the function of real balances is solely to reduce the transactions costs of acquiring final goods, these balances enter as an argument in the indirect utility function as a proxy for the transactions costs saved, rather than as a final good. In this case, differential tax theory indicates that intermediate goods should not be taxed.[4] Second, our analysis is limited to the case of perfectly anticipated inflation and focuses solely on the welfare loss due to the reduction of real balances. It does not capture the other costs of inflation, nor is it addressed to the realistic case of a less than fully anticipated inflation.[5]

LITERATURE CITED

Aurenheimer, Leonardo. "The Honest Government's Guide to the Revenue from the Creation of Money." *Journal of Political Economy* 82 (May/June 1974), 598–606.

Bailey, Martin J. "The Welfare Cost of Inflationary Finance." *Journal of Political Economy* 64 (April 1956), 93–110.

Calvo, Guillermo, and Rogue Fernandez. "Competitive Banks and the Inflation Tax." *Economic Letters* 12 (1983), 313–17.

Dwyer, Gerald P., Jr., and Thomas R. Saving. "Government Revenue from Money Creation with Government and Private Money." *Journal of Monetary Economics* 17 (March 1986), 239–49.

Kimbrough, Kent P. "Inflation, Employment, and Welfare in the Presence of Transactions Costs." *Journal of Money, Credit, and Banking* 18 (May 1986), 127–40.

Lucas, Robert E., Jr. "Principles of Fiscal and Monetary Policy." *Journal of Monetary Economics* 17 (January 1986), 117–34.

Lucas, Robert E., Jr., and Nancy L. Stokey. "Optimal Fiscal and Monetary Policy in an Economy without Capital." *Journal of Monetary Economics* 12 (January 1983), 55–93.

Marty, Alvin L. "A Note on the Welfare Cost of Money Creation." *Journal of Monetary Economics* 2 (January 1876), 121–24.

———. "Inflation, Taxes, and the Public Debt." *Journal of Money, Credit, and Banking*" 4 (November 1978), 437–52.

McClure, J. Harold, Jr. "Welfare-Maximizing Inflation Rates under Fractional Reserve Banking with and without Deposit Rate Ceilings." *Journal of Money, Credit, and Banking* 18 (May 1986), 233–38.

Phelps, Edmund S. "Inflation in a Theory of Public Finance." *Swedish Economic Journal* 75 (1973), 67–82.

[4]See, for example, Lucas and Stokey (1983), Kimbrough (1986), and Lucas (1986).

[5]We might relax the assumption of fully anticipated inflation and let the expected rate equal the actual rate with a constant variance. But, what of the case in which the variances of the actual and expected rates of inflation increase with the means of these rates. In this case, the traditional analysis of the welfare cost is not applicable